Radiation Detection and Mapping Using Mobile Robots

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Motivation for nuclear detection

- Emerging nuclear threats
- Surveillance and monitoring of nuclear waste/storage facilities
- Nuclear nonproliferation
- Lack of technology to address situational awareness

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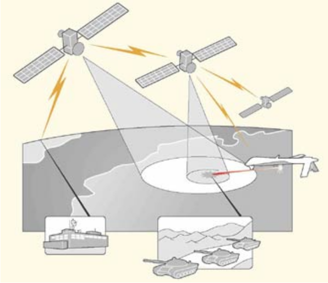
Needle in a haystack

Problems

- Autonomously detect radiation sources generating weak signals burried within a sea of background events
- Autonomously map radiation levels over a given area to provide real-time situational awareness

Prior work: mobile sensor networks

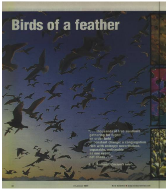
- Groups of sensors used to monitor, track, and survey [Oh et al., 2006]
- Mobile and stationary sensors for surveillance [Huntwork et al., 2006]
- Gradient climbing for distribution of sensors [Cortes et al., 2005, Orgen et al., 2004]



source: (LANL) "vision of future response to global threats "

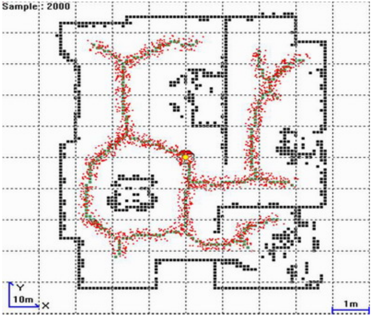
Prior work: multi-agent coordination

- Flocking [Tanner et al., 2007, Olfati-Saber, 2006]
- Swarming [Gazi and Passino, 2002]
- Pursuit-evasion games [Antoniades et al., 2003, Yamaguchi, 1998]
- Formation keeping [Das et al., 2002, Tanner and Kumar, 2005]
- Geometric optimization [Cortes et al., 2005]



Prior work: topological mapping

- Simultaneous Localization and Mapping [Dissanayake et al., 2001]
- Three dimensional maps in dynamic environments [Hahnel et al., 2003]
- Occupancy grids [Elfes and Moravec, 1985]
- Frontier based exploration [Yamauchi, 1998, Thrun, 2001]



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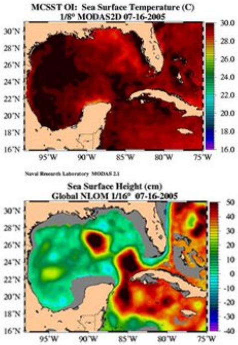
Prior work: spatial distribution mapping

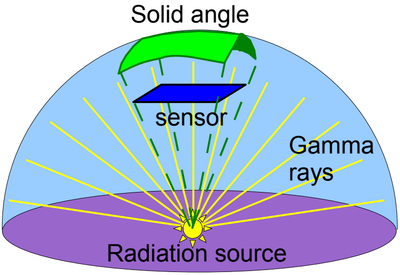
- Mapping gas concentrations [Lilienthal and Duckett, 2004]
- Tracking ocean features [Orgen et al., 2004]
- Mapping probability of detection [Kim and Hespanha, 2004, Bertuccelli and How, 2005]



source: Pelagic Fisheries Conservation Program - Texas A&M University at Galveston

How to find the needle

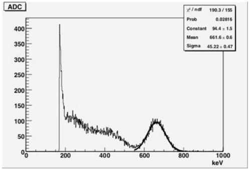
- Increase detector area: background signal $\propto$ area; source signal $\propto$ solid angle
- Design better sensors: increase resolution
- Bring a smaller sensor close to source: for same signal, area $\propto R^{-2}$

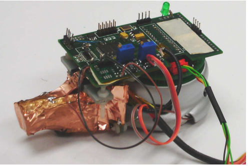


The mobile sensor

CsI sensor → Charge-sensing preamplifier → Shaping amplifier → 8bit ADC → FPGA → Robot

CsI sensor	Charge-sensing preamplifier	Shaping amplifier	8bit ADC	FPGA	Robot
AlphaSpectra CsI crystal Hamamamtsu PD 3509	Low noise Amptek A250	4-stage shaping (design by R. Schirato, P-25)	National Semiconductor Low power, High-Speed, 8bit ADC08200	Altera Cyclone 2910, 65 IOs	Khepera II miniature mobile robot by K-Team Inc





Sequential testing

- Single pass, tuned to maximum expected background and minimum source activity
- Confirms or rejects existence of radioactive material
- No information about background radiation levels
- Sensitive to assumption of source strength

$$P(N|S) = \frac{(t \cdot \mu_s)^N}{N!} e^{-t \cdot \mu_s}$$

$$P(N|B) = \frac{(t \cdot \mu_b)^N}{N!} e^{-t \cdot \mu_b}$$

$$\kappa = \frac{P(N|S)}{P(N|B)}$$

$$\kappa < \frac{P_{fn}}{1 - P_{fa}} \text{ OR } \kappa > \frac{1 - P_{fn}}{P_{fa}}$$

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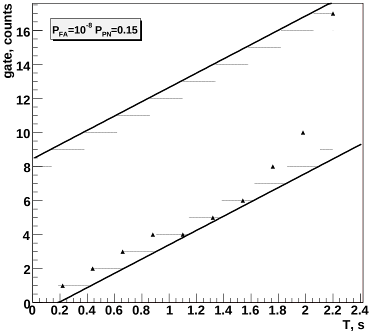
Robotic sequential search [Cortez et al., 2007b]



Control velocity to regulate exposure :

$$a[k] = \frac{2(L - v[k]T_t)}{\Delta(2(T_t - T_p) - \Delta)}$$

Bayesian mapping [Cortez et al., 2007a]

- Gamma distribution on each cell

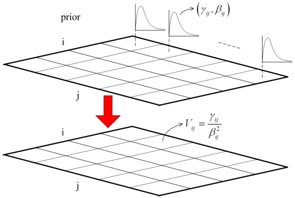
$$\pi(\lambda) = \beta^{\gamma}\lambda^{\gamma-1}e^{-\lambda\beta}\Gamma(\gamma)^{-1}$$

- Expectation

$$\mathbb{E}(\lambda) = \frac{\gamma}{\beta}, \quad V(\lambda) = \frac{\gamma}{\beta^2}$$

- Reduce uncertainty (variance) in radiation distribution

$$\gamma_{ij}^{+} = \gamma_{ij} + c \qquad \beta_{ij}^{+} = \beta_{ij} + 1$$



Bayesian update

Assumptions

- $P(c,t) = \frac{(\mu \cdot t)^c}{c!} e^{-(\mu \cdot t)}$
  $\mu = \chi \cdot \alpha \int_0^t \frac{1}{r^2(t)}\, dt$
- Probability of registering c counts:

$$f(c) = f(c|\alpha, \chi, r(t))$$

- Radiation prior

$$f(\alpha) = \begin{cases} \frac{1}{\alpha_2 - \alpha_1}, & \text{if } \alpha_1 < \alpha < \alpha_2 \\ 0, & \text{otherwise} \end{cases}$$

- $f_c(c) =$

$$\int_{\alpha_1}^{\alpha_2} \left[\frac{(\chi \cdot \alpha \int_0^t \frac{1}{r^2(t)}\, dt)^c}{c!} \cdot e^{-(\chi \cdot \alpha \int_0^t \frac{1}{r^2(t)}\, dt)} \right] d\alpha$$

$$\boxed{f(\alpha|c, \chi, r(t)) = \frac{f(\alpha) \cdot f(c|\alpha, \chi, r(t))}{f_c(c)} \cdot \sigma}$$

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Sensor: a communication channel

Differential Entropy

quantifies the amount of information gained about the world by a radiation measurement

$$h(A|C) = -\int_{\alpha_1}^{\alpha_2} f(\alpha|c) \cdot \log_2 f(\alpha|c)\, d\alpha$$

Mutual Information

- $I(A;C) = h(A) - h(A|C)$
- $I(A;C) \geq 0$ with equality iff A and C are independent

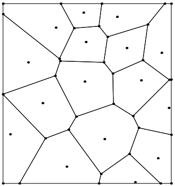
Information surfing

is about moving the sensor along $\nabla I(A;C)$ to locally maximize mutual information

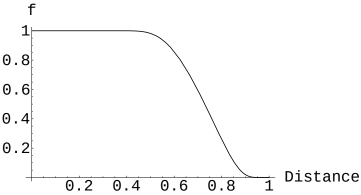
Area Partitioning and Sensor Performance

- Dynamic Voronoi Partitioning [Cortes et al., 2005]

$$V_i(P) = \{q \in Q \mid \|q - p_i\| \leq \|q - p_j\|, \forall p_j \in P\}$$



- Sensing Performance Function



$$f(\|q-p_i\|)=\begin{cases} \dfrac{\exp\left(\frac{-1}{(R^2-\|q-p_i\|)}\right)}{\exp\left(\frac{-1}{(R^2-\|q-p_i\|)}\right)+\exp\left(\frac{-1}{(\|q-p_i\|-R^2)}\right)} \\ 1 \\ 0 \end{cases}$$

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Control Design

Objective Function

- Cortes & Bullo approach [Cortes et al., 2005]

$$\mathcal{H}_i(P) = \int_{V_i(P)} f(\|q - p_i\|)\phi(q)\, dq$$

- Our approach

$$\mathcal{W}_i(P) = \int_{V_i(P)} f(\|q - p_i\|)I(q,p,t)\, dq$$

Control design

$$\dot{p}_i = \left[\int_{V_i(P)} \frac{\partial f(\|q-p_i\|)}{\partial p_i} I(q,p,t)dq + \int_{V_i(P)} f(\|q-p_i\|)\frac{\partial I(q,p,t)}{\partial p_i} dq \right]$$

Stability Analysis

Challenge

Closed loop system is time varying; the invariance principle argument used in [Cortes et al., 2005] does not apply.

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Lemma

$$\lim_{t \to \infty} \frac{\partial I(q,p,t)}{\partial t} = 0.$$

Stability Analysis

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Lemma

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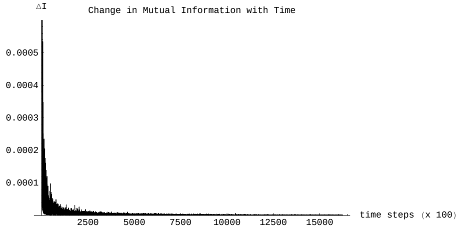

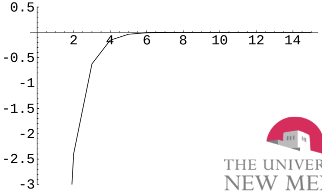

Outline of Proof

- $\frac{\partial I(q,p,t)}{\partial t} = \frac{\partial h}{\partial d}\frac{\partial d}{\partial t}, \;\; d \triangleq \int_0^t \frac{1}{r^2(\tau)}\, d\tau$
- $\frac{\partial h}{\partial d} = \sum_{i=1}^{12} A_i$
- $A_1 \triangleq \frac{(-1-c)\Gamma(1+c,\alpha_1 \cdot d \cdot \chi)^2}{(1+c) d \cdot c!(\Gamma(1+c,\alpha_1 \cdot d \cdot \chi) - \Gamma(1+c,\alpha_1 \cdot d \cdot \chi)^2)) \log 2}$
- $\lim_{d\to\infty} A_1 = 0$
- $c!$ changes exponentially fast whereas d changes linearly
- $A_9 \triangleq \frac{\alpha_1^2 c d e^{(-\alpha_1-\alpha_2)d\chi + \alpha_2 d\chi}\chi^2(\alpha_1 d\chi)^{2c}\Omega(1+c,1+c,2+c,2+c,-\alpha_1 d\chi)}{(1+c)^2(\Gamma(1+c,\alpha_1 d\chi) - \Gamma(1+c,\alpha_2 d\chi))^2 \log 2}$
- $\Omega(\cdot) = \sum_{n=0}^{\infty} \frac{(1+c)_n(1+c)_n}{(2+c)_n(2+c)_n} \cdot \frac{-\alpha_1^n}{n!}$
- $\Omega(\cdot) = 0$

$$\therefore \lim_{d\to\infty} \frac{\partial h}{\partial d} = 0 \implies \lim_{t\to\infty} \frac{\partial I(q,p,t)}{\partial t} = 0$$

Stability Analysis

Proposition

Consider the gradient field defined by the control input u_i. Then the system stabilizes at configurations that (locally) minimizes the information flow from each robot, as expressed by the product $I(q,p,t)f(\|q-p_i\|)$, for $i=1,\ldots,n$.

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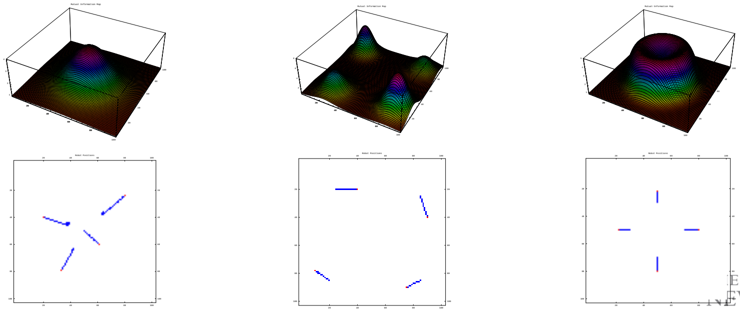

Outline of Proof

- $\frac{\partial(I(q,p,t)f(\|q-p_i\|))}{\partial p_i} = \frac{\partial f(\|q-p_i\|)}{\partial p_i} I(q,p,t) + f(\|q-p_i\|)\frac{\partial I(q,p,t)}{\partial p_i}$
- Lyapunov function candidate:
  $\mathcal{W}(P) = (\sum_i \int_{V_i(P)} f(\|q-p_i\|) I(q,p,t)\, dq)^{-1}$
- $\dot{\mathcal{W}}(P) = \sum_i \frac{\partial \mathcal{W}(P)}{\partial p_i}\dot{p}_i + \frac{\partial \mathcal{W}(P)}{\partial t}$
- $\dot{\mathcal{W}}(P) = -\mathcal{W}(P)^2 \sum_i \Big[\int_{V_i(P)} \frac{\partial f(\|q-p_i\|)}{\partial p_i} I(q,p,t) dq + \int_{V_i(P)} f(\|q-p_i\|) \frac{\partial I(q,p,t)}{\partial p_i} dq \Big]^2 - \mathcal{W}(P)^2 \sum_i \int_{V_i(P)} f(\|q-p_i\|) \frac{\partial I(q,p,t)}{\partial t} dq$
- If it does not stabilize at configurations $\frac{\partial I(q,p,t) f(\|q-p_i\|)}{\partial p_i} = 0$,

The contradiction

- $\left| \frac{\partial(I(q,p,t)f(\|q-p_i\|)}{\partial p_i} \right| > \epsilon \quad \forall t > T$
- $W_3(\epsilon) \triangleq \mathcal{W}(P)^2 \sum_i \int_{V_i(P)} \epsilon^2 dq$
  $\beta(t) \triangleq -\mathcal{W}(P)^2 \sum_i \int_{V_i(P)} f(\|q - p_i\|) \frac{\partial I(q,p,t)}{\partial t} dq$
- $\dot{\mathcal{W}}(P) \leq -W_3(\epsilon) + \beta(t) \leq -(1-\theta)W_3(\epsilon) - \theta W_3(\epsilon) + \beta(t)$, where $0 < \theta < 1$
- $\lim_{t\to\infty} \frac{\partial I(q,p,t)}{\partial t} = 0 \implies \lim_{t\to\infty} \int_{V_i(P)} f(\|q - p_i\|) \frac{\partial I(q,p,t)}{\partial t} dq = 0$
- After suffiecent time, τ, $-\theta W_3(\epsilon) + \beta(t) \leq 0 \quad \forall t > \tau$
  $\implies \dot{\mathcal{W}}(P) \leq -(1-\theta)W_3(\epsilon) \triangleq \dot{\gamma}(t) \implies \gamma(t) = \gamma(0) - (1-\theta)W_3(\epsilon)t$
- Comparison Lemma gives
  $\mathcal{W}(P) \leq \gamma(t) = \gamma(0) - (1-\theta)W_3(\epsilon)t \implies \mathcal{W}(P) < 0$
- But by construction $\mathcal{W}(P) \geq 0$!

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Implementation issues

Ideally

For each point p, and total counts c collected there, update $f(\alpha|c,\chi,r)$ through Bayes rule.

Issues

- Numerical instabilities: the incomplete gamma function $\Gamma(1+c,\alpha_1 d\chi)$
- The assumption on uniform prior: $f(\alpha) = \frac{1}{\alpha_2 - \alpha_1}$
- Convergence speed

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Algorithm implementation

- Outline of Algorithm
  - Area discretized into 100 x 100 cell grid, initially uniform $f(\alpha)$
  - Five time periods for each measurement cycle
  - Update the total counts collected at cell (i,j): $\bar{c}^{+}_{i,j} = \frac{c_{i,j}\cdot(M_{i,j}-1)+\bar{c}_{i,j}}{M_{i,j}}$
  - Update uniform $f(\alpha)$ at cell (i,j): $\delta \triangleq \min\{|\frac{\bar{c}_{i,j}}{\chi d} - \alpha_1|, |\frac{\bar{c}_{i,j}}{\chi d} - \alpha_2|\}$; $\alpha_1^{+} = \frac{\bar{c}_{i,j}}{\chi d} - \delta$, $\alpha_2^{+} = \frac{\bar{c}_{i,j}}{\chi d} + \delta$
  - Speed up by updating neighborhood: $c_{i,j} = \left\lfloor \frac{c}{\chi d_{i,j}} \right\rfloor$
    - How probable is this "remote" count? $F = \frac{f(\alpha)\cdot f(\bar{c}_{i,j}|\alpha,\chi,r(t))}{f_c(\bar{c}_{i,j})} \cdot \sigma$
    - New candidate for $\alpha_{i,j}$ $\alpha' = \frac{\frac{1}{\alpha_2-\alpha_1}\cdot\frac{\alpha 1+\alpha 2}{2}+F\cdot c_{i,j}}{\frac{1}{\alpha_2-\alpha_1}+F}$
    - Is that candidate better? $F' = \frac{f(\alpha')\cdot f(\bar{c}_{i,j}|\alpha',\chi,r(t))}{f_c(\bar{c}_{i,j})} \cdot \sigma$
    - If $F < F'$, update $\alpha^{+} = \alpha'$,
      and bounds $\alpha_1^{+} = \alpha - \frac{1}{2\cdot F'}$, $\alpha_2^{+} = \alpha + \frac{1}{2\cdot F'}$

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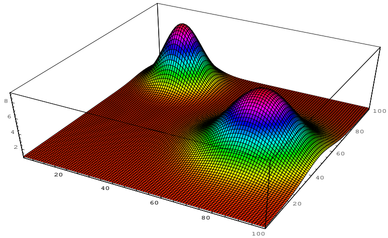
Simulation Results

Radiation Map



Mutual Information Map

Average difference between real and estimated map is .42 counts/sec

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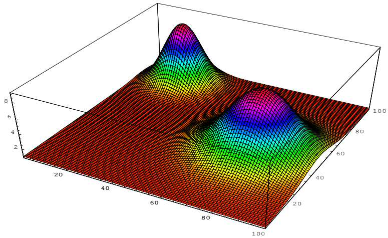

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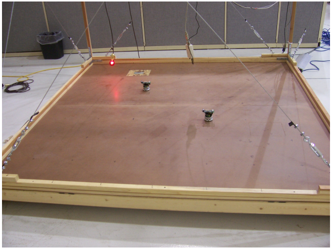
Experimental results

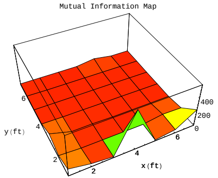
- Khepera II robot: Wireless RF turret
- RADDS multi-robot coordination software environment
- Light intensity instead of radiation
  - Poisson filtering of IR readings
- Crossbow Cricket network for localization
  - Interference issues with wireless data transmission



Experimental Results (cont'd)

Experimental Test

Mutual Information



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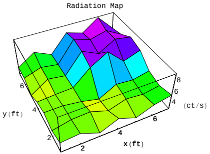

Conclusion

Outcomes

- Cooperative coordination of robot team for radiation mapping
- Decentralized execution
- Stable and fault tolerant

Open issues

- Vehicle dynamics
- Robot localization
- Complete decentralization
- Obstacle avoidance

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Lessons from flocking

Local interaction

$$u_i = -\sum_{j \in \mathcal{N}_i} (v_i - v_j) - \sum_{j \in \mathcal{N}_i} \nabla_{r_i} V_{ij}(\|r_{ij}\|)$$

Force symmetry and Lyapunov stability

$$W(\bar{r}, v) = \frac{1}{2} \sum_{i=1}^{N} (V_i + v_i^T v_i) \Rightarrow \dot{W} = \sum_{i=1}^{N} v_i^T \nabla_{r_i} V_i - v^T L v - \sum_{i=1}^{N} v_i^T \nabla_{r_i} V_i$$

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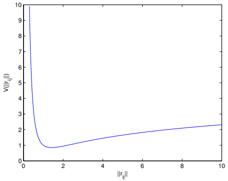

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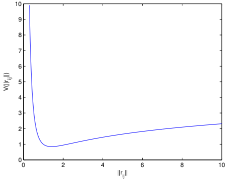

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Need a different approach

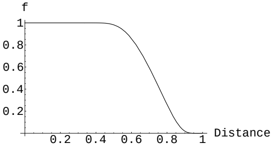
Why it will not work here

- The canceling terms result from "kinetic" + "potential"
- In the mapping problem: different Lyapunov function

Alternatives

Make collision configurations "uninteresting"

$$I(q,p,t) \prod_{i,j \in \{1,\dots,N\}} \varphi(\|p_i - p_j\|)$$



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- DoE URPR Grant: DE-FG52-04NA25590
- . . . The Knack

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